A Partial Analysis of the Mechanics of the Cross Country Navigation Problem

R. Craig Coulter

CMU-RI-TR-94-02

The Robotics Institute

Carnegie Mellon University

Pittsburgh, Pennsylvania 15213

January 12, 1994

This research was sponsored by ARPA, under contracts "Perception for Outdoor Navigation" (contract number DACA76-89-C-0014, monitored by the US Army TEC) and "Unmanned Ground Vehicle System" (contract number DAAE07-90-C-R059, monitored by TACOM). Views and conclusions contained in this document are those of the author and should not be interpreted as representing official policies, either expressed or implied, of ARPA or the United States Government.

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*

2. REPORT DATE
January 12, 1994

3. REPORT TYPE AND DATES COVERED
technical

4. TITLE AND SUBTITLE
A Partial Analysis of the Mechanics of the Cross Country Navigation Problem

5. FUNDING NUMBERS
DACA-89-C 0014
DAAE07-90 [illegible]2059

6. AUTHOR(S)
R. Craig Coulter

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
The Robotics Institute
Carnegie Mellon University
Pittsburgh, PA 15213

8. PERFORMING ORGANIZATION REPORT NUMBER
CMU-RI-TR-94-02

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
ARPA

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release;
Distribution unlimited

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*
This report investigates the nature of the externally applied forces on a mobile robot traversing natural terrain. A set of relative metrics is derived which are useful in the design of a powertrain controller for such a robot. These metrics are applied to a specific example case, the NavLab II, operating in rolling offroad terrain.

14. SUBJECT TERMS

15. NUMBER OF PAGES
2[illegible]

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
unlimited	unlimited	unlimited	unlimited

Abstract

This report investigates the nature of the externally applied forces on a mobile robot traversing natural terrain. A set of relative metrics is derived which are useful in the design of a powertrain controller for such a robot. These metrics are applied to a specific example case, the NavLab II, operating in rolling offroad terrain.

Table of Contents

List of Figures

1.0 Introduction

Control is the problem of producing a desired dynamic state in a physical system. *Motion control* is the particular problem of controlling the physical positions and velocities of a mechanical system. This is often thought of as the *controlled alteration* of a system's natural dynamic response to an input. The implication is that, in absence of control, the system would produce an undesired dynamic response in response to these inputs. In the case of a mechanical system, it is clear that the agents producing the dynamic response are the forces applied to the system. From classical mechanics, it is well known that the motion of a body is governed by the forces applied to that body. From the point of view of mechanics, control may be seen as the application of a *control force* to alter the motion to a desired *control motion*. In the context of control theory, these externally applied forces are called *disturbances*, as they tend to disturb the system from a desired state.

The problem of motion control in outdoor navigation is complicated by the presence of disturbance forces whose magnitude is significant when compared to the control force. This paper investigates the origins of these disturbances, and seeks to quantify their magnitudes into a set of force regimes - conditions under which one or another of the disturbances is most dominant. A set of relative analysis techniques is derived that compares the magnitude of the forces to the state of the vehicle, providing a means for quantitatively assessing the *significance* of a disturbance to the system. The techniques are applied to a specific problem, the terrestrial navigation of a large truck over rugged terrain as a precursor for the design of an enhanced autonomous motion controller.

The autonomous control or autonomous navigation of a mobile robot is achieved by a system that recognizes and avoids obstacles in the mobile robot's path. Traditionally, the concept of an obstacle has been limited to discrete solid objects such as rocks and trees, or more generally, as poses or stances of the robot that cause it to be incapable of continuing progress. As autonomous navigation systems become more capable, more enhanced motion control is required to enable them to drive in more aggressive environments. Forces that were previously unimportant to planners and controllers have now become more important. This paper implies that the notion of an obstacle can be expanded to included any entity that can impress a force on the mobile robot retarding or impeding its progress.

2.0 Elementary Mechanics - Externally Applied Forces

In this section, forces and force rates that will be used later in the analysis section of the report are derived. Consider the NavLab II mobile robot as a single lumped mass, acted upon by a set of forces, as shown in the following diagram. The forces are produced by one of three means:

- Forces arising from the gravitational field.
- Forces arising from the motion of the machine in a fluid field.
- Forces arising in reaction to the action of the machine upon the soil.

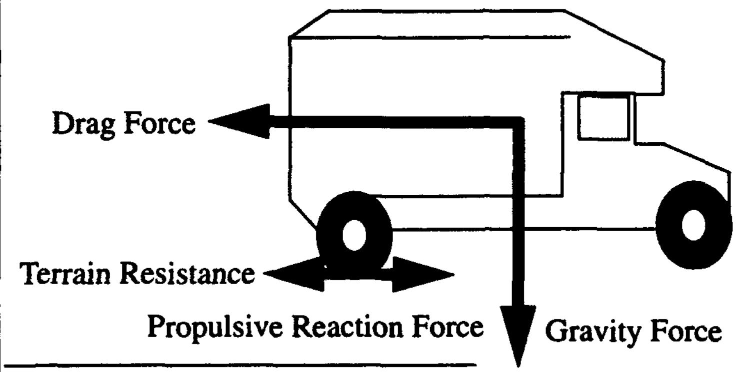

Figure 1: Externally Applied Forces

Note the dissimilarity in nature among the forces: the gravitational force is a *field force*, the drag force arises from *viscous friction*, and the terrain resistance arises from *mechanical friction*.

2.1. Gravity Analysis

The combined force of gravitational attraction and the Coriolis effect as felt at the Earth's surface is known as the *local gravity force*. Gravity, like gravitation, is treated as a field force. It is represented locally by the vector **g**. The magnitude and direction of this force are constant, making its effects upon the vehicle a *function of the orientation*.[1]

In conventional vehicles, there are two principal gravitational effects:

- Gravity loading of the powertrain.
- Gravity loading in the static stability constraint polygon.

The projection of the gravity vector on the vehicle's propulsion axis manifests itself as a *load on the powertrain*. Clearly, this is of concern to vehicle motion control. Gravity loading in the prediction of static stability is of concern in motion planning, and will not be considered in this report.

2.1.1 Powertrain Loading

The gravitational load that acts on the powertrain is found to be:

$$\boxed{F_g = mg\sin\Theta}$$

1. Note that the value of the Coriolis force varies with latitude, and the magnitude of gravitational attraction varies with altitude, so the magnitude of g is only locally constant.

where Θ is the pitch of the vehicle.

2.1.2 Rate of Change of Powertrain Load

The time rate of change of this force is given by:

$$\frac{dF_g}{dt} = mg\cos\Theta\frac{d\Theta}{dt} = mgv\cos\Theta\frac{d\Theta}{ds}$$

where Θ is the pitch of the vehicle and s measures the distance along the vehicle's path. The time derivative of s is represented by v - the vehicle's speed.

2.2 Aerodynamic Analysis

The relative motion of the vehicle and its surrounding fluid gives rise to a set of three aerodynamic forces: *lift*, *drag* and *side* force; and a moment about each axis. For the purposes of this analysis, it is assumed that any winds that the vehicle encounters will be small (< 10 m.p.h.), thus the principal aerodynamic force encountered will be the drag force.[1]

2.2.1 Drag Force

The force of viscous friction at the air / vehicle surface acts to retard vehicle motion. This viscous friction is commonly called the *drag force*. The drag force has two sources: *form drag* and *friction drag*. Form drag is the result of the pressure differential formed as the vehicle body passes through the fluid. The pressure P_1 in front of the vehicle is higher than the pressure P_2 behind the vehicle, which gives rise to a retarding force $(P_1 - P_2) * A$, where A is the effective cross sectional area. Friction drag results from the contact between the fluid stream and the vehicle surface.

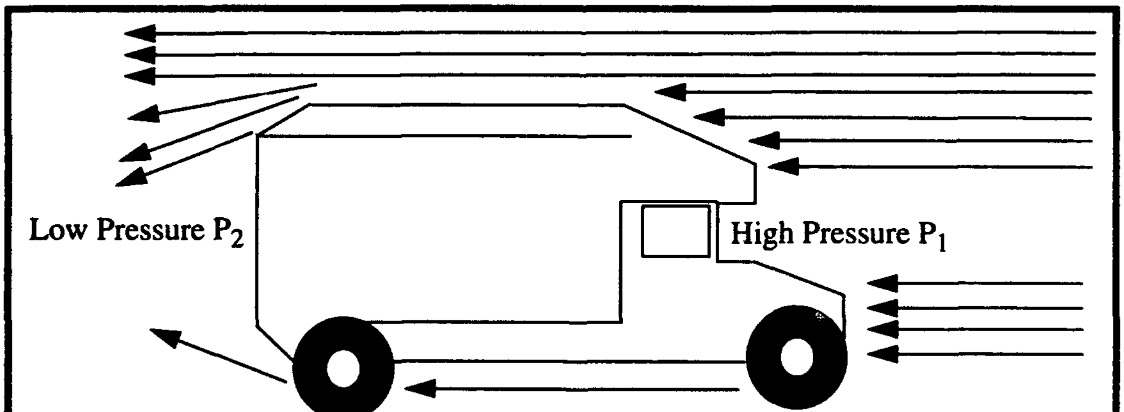

Figure 2: Aerodynamic Drag

The total drag force is empirically represented by the following formula:

$$F_d = \frac{1}{2}\rho V^2 C_d A$$

1. If this assumption turns out to be untrue, it is interesting to note that the formulation of force equations for the other forces is nearly identical to that of the drag force, which is presented herein.

where ρ represents the density of air, A is the effective cross sectional area of the vehicle, V is the velocity of the vehicle ***relative to the air***, and C_d is the drag coefficient.

2.2.2 Drag Force Sensitivity

The sensitivity of the drag force to changes in vehicle speed is found by taking the first derivative of F_d with respect to V.

$$\boxed{\frac{dF_d}{dt} = (\rho C_d A) V \frac{dV}{dt}}$$

2.3 Rolling Resistance Analysis

The interaction of the vehicle tires with the terrain results in an energy loss, most commonly referred to as rolling resistance. There are many models of rolling resistance. As a first order approximation, I will use a coulombic model.

$$\boxed{F_R = fmg}$$

where f is the rolling resistance coefficient. Changes in mechanical configuration, such as tread design, tire inflation, soil compaction, and tire temperature can and does affect the value of the resistance coefficient. However, the rate of change of these variables is often insignificant to the control problem at hand. Rolling resistance is also a function of vehicle speed, though the sensitivity is small below speeds of ~ 80 - 100 m.p.h. (Gillespie[1])

2.4 Orthogonality of the Forms of the Forces

Each of the forces is parameterized in a nearly orthogonal space. The gravity load is a function of terrain geometry and vehicle mass. The aerodynamic load is a function of fluid properties, vehicle geometry and the speed of the vehicle. Rolling resistance is a function of the tire and soil properties, and the mass of the vehicle. The forces are generally orthogonal in their parameterization, the only exception being that both mechanical friction and gravity forces are mass dependent.

$$\boxed{\begin{gathered} F_g = fn(m, \Theta) \\ F_a = fn(\rho, C_d, A, V) \\ F_r = fn(m, f) \end{gathered}}$$

3.0 Relative Analysis of the External Forces

This section examines the *implication* of a force on a mass. When does the result of an applied force become significant? This question is of some importance because it allows an engineer to determine, before a control design is undertaken, which forces are significant enough to warrant compensation. Using Newton's laws of motion, a set of quantitative expression is now derived which describe the significance of an impressed force.

3.1 Representations of Motion

The motion of a body is described by Newton's laws. However, Newton's laws may be expressed in a number of different forms. Newton's second law, in its original form, is written:

$$\boxed{\Sigma\vec{F} = \frac{d\vec{p}}{dt}}$$

The *impressed forces* act to change what Newton called the *quantity of the motion*, here represented by p, and more commonly called the *momentum*. Integrating both sides with respect to time leads to a new form, which is called the ***impulse-momentum form of Newton's second law***:

$$\boxed{\vec{p} = \int(\Sigma\vec{F})\,dt = \Sigma\int\vec{F}dt}$$

The terms on the right, which are the time integrals of the impressed forces are called *impulses*. If each impulse is denoted J_i, then we can more compactly express this form as:

$$\boxed{\vec{p} = \Sigma\vec{J}}$$

The impulse momentum equation considers the result of the impression of a force *over time*. A third expression, called the ***work-energy form of Newton's second law***, can be obtained by considering the impression of a force *through a distance*. Starting with Newton's second law, rewrite p as the product of the mass and the velocity, mv, and assume that the mass is constant.

$$\boxed{\Sigma\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt}}$$

Integrate both sides through the distance dx.

$$\boxed{\int(\Sigma\vec{F})\,dx = \Sigma\int\vec{F}dx = \int m\frac{d\vec{v}}{dt}dx}$$

Realize that **F** and **v** have the same vector direction, along the line of integration, which would allow us to reduce the vector integration to a scalar one. This equation is written with a summation; it is only the resultant force of that summation that has the same vector direction as the resultant velocity of the mass. In order to preserve the

summation, invoke the principle of superposition, and realize that this integration can be performed for each of the impressed forces and then summed. The summation and the integral can be interchanged.

$$\boxed{\Sigma\int F dx = \int m\frac{dv}{dt}dx}$$

The term on the left, in the previous equation, is defined to be the *work done by the force F*. The terms on the right integrate to the *change in kinetic energy of the mass*.

$$\Sigma W = \int m\frac{dv}{dt}dx = m\int\frac{dx}{dt}dv = m\int v dv$$

$$\Sigma W = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

This work-energy relationship is unique among the three forms because it is the only scalar representation of the result of the action of an impressed force. It describes the result of a force acting through a distance as a change in the mass property called the *kinetic energy*.

3.2 Relative Representations

In the previous section three forms of Newton's second law were derived. In this section each form a *normalized* form of each expression is derived.

3.2.1 Relative Force and Relative Strength

Newton's second law can be normalized by a particular force F_i. The sum of the forces is equal to the total force, denoted F_t. If the individual forces are normalized by the total force, a set of dimensionless force ratios follows:

$$F_1 + F_2 + F_3 + \ldots = F_t \qquad N_i = \frac{F_i}{F_t}$$

$$\frac{F_1}{F_t} + \frac{F_2}{F_t} + \frac{F_3}{F_t} + \ldots = 1 \qquad N_1 + N_2 + N_3 + \ldots = 1$$

In a similar vein, let the sum of the absolute value of the forces be called the total strength and denoted S_t. Let the ratio of a force magnitude to the total strength be called the *relative strength*, S:

$$|F_1| + |F_2| + |F_3| + \ldots = S_t \qquad S_i = \frac{|F_i|}{S_t}$$

$$\frac{|F_1|}{S_t} + \frac{|F_2|}{S_t} + \frac{|F_3|}{S_t} + \ldots = 1 \qquad S_1 + S_2 + S_3 + \ldots = 1$$

3.2.2 Relative Impulse Representation

Using the impulse-momentum relationship, the effect of an applied force through time can be examined. The change in momentum of a force F, applied through time is equal to the impulse J:

$$\boxed{\vec{J} = \int \vec{F} dt}$$

This impulse may be normalized by the momentum of the body. Call this the *normalized impulse*, and define it to be:

$$\boxed{\vec{J}_n = \frac{\int \vec{F} dt}{mv}}$$

3.2.3 Relative Energy Representation

The work-kinetic energy relationship quantifies the effect of a force applied through a distance. The change in the kinetic energy of a body produced by a force F, applied through a distance is equal to the work W:

$$\boxed{\Delta KE = W = \int F dx}$$

This work can be normalized by the kinetic energy of the body. Call this the *normalized work*, and define it to be:

$$\boxed{W_n = \frac{\int F dx}{\frac{1}{2} m v^2} = \frac{2 \int F dx}{m v^2}}$$

3.2.4 Summary

Each of the three relative representations offers a different perspective on the results of an application of a force on a body. The ***relative strength of a force*** can be used to find regimes in which particular forces dominate the body. This is especially useful when designing a control system, as it allows the engineer to determine which forces may require special attention, as opposed to those that can be treated as random disturbances. The ***normalized momentum*** is useful in the analysis of the implication of an "obstacle" during navigation. If the notion of an obstacle is expanded from a solid physical object, like a brick wall, to anything that applies a force that tends to impede progress, then the ***significance*** of the obstacle relative to the momentum of our vehicle can be calculated. This will be shown in more detail in the next section. The ***normalized kinetic energy*** similarly allows a useful analysis of an obstacle. The obvious difference between the normalized momentum and the normalized kinetic energy representations is that one considers a force moved through a distance, while the other considers a force applied through time.

3.3 Relative Strength

In this section, the general formulations will be applied to the problem using the three externally applied forces from the second section. Recall that it has been shown that the externally applied forces take the following form:

$$F_r = fmg \qquad F_g = mg\sin\Theta \qquad F_d = \frac{1}{2}\rho V^2 C_d A$$

3.3.1 Relative Strength of the Rolling Resistance

The relative strength of the rolling resistance is found to be:

$$S_r = \frac{fmg}{mg\,(f+|\sin\Theta|)+\frac{1}{2}\rho V^2 C_d A} = \frac{f}{f+|\sin\Theta|+\frac{\rho V^2 C_d A}{2mg}}$$

To simplify the denominator, define a constant γ, and call it the *relative strength coefficient.*

$$\gamma = \frac{\rho C_d A}{2mg}$$

$$S_r = \frac{f}{f+|\sin\Theta|+\gamma V^2}$$

3.3.2 Relative Strength of the Gravity Load

The relative strength of the gravity load is found to be:

$$S_g = \frac{|\sin\Theta|}{f+|\sin\Theta|+\gamma V^2}$$

3.3.3 Relative Strength of the Drag Load

The relative strength of the drag load is found to be:

$$S_d = \frac{\gamma V^2}{f+|\sin\Theta|+\gamma V^2}$$

3.4 Relative Impulse Analysis

3.4.1 Relative Impulse of the Gravity Load

Gravity loads are caused by terrain deviations. Large, long terrain deviations are things like hills, while small terrain deviations are things like rocks and potholes. A constant terrain deviation can be characterized by its angle of inclination Θ and its length L. Assuming that the deviation is to be traversed at the current velocity V, it will do so in time $\Delta t = L/V$. Using this relationship, solve for the relative impulse of a gravity load as follows:

$$\boxed{I_g = \frac{mg\sin\Theta\,(\Delta t)}{mV} = \frac{g\sin\Theta\,(\Delta t)}{V} = \frac{g\sin\Theta L}{V^2}}$$

3.4.2 Relative Impulse of the Drag Force

The drag load is caused by the motion of the body through a viscous fluid, in this case air. After writing the relative impulse, note that a substitution can be made using the non-dimensional term γ, as well as a substitution of the distance travelled L for the product $V\Delta t$.

$$\boxed{I_d = \frac{\frac{1}{2}\rho V^2 C_d A\,(\Delta t)}{mV} = \gamma g L}$$

This particular ratio is interesting because it is *not dependent upon the speed at which the vehicle travels, or the time of traversal*, but only on the distance travelled. This seems counter-intuitive, especially considering that the drag force is a function of velocity. You can convince yourself of the truth of the final ratio by considering that the drag force rises quadratically in velocity, while the momentum rises linearly, yielding a linear ratio. Since the definition of impulse requires multiplication by the elapsed time, an expression of distance results. For any distance travelled, if travelled at a higher speed, less time is needed. Thus while the ratio of the drag force to the momentum increases linearly, the elapsed time decreases linearly.

3.4.3 Relative Impulse of the Rolling Resistance

Rolling resistance is caused by the interaction of the vehicle tires with the soil, neglecting the work done in compacting or bulldozing soil. Writing the relative impulse and substituting, as before, for Δt produces the following ratio:

$$\boxed{I_r = \frac{fmg\,(\Delta t)}{mV} = \frac{fgL}{V^2}}$$

3.4.4 Summed Relative Impulse

Summing the individual relative impulses, leads to the following expression.

$$\boxed{I_T = I_d + I_g + I_r = \gamma g L + \frac{g\sin\Theta L}{V^2} + \frac{fgL}{V^2}}$$

Grouping terms and defining the parameter λ, yields:

$$I_T = \gamma g L + \frac{gL}{V^2}(f + \sin\Theta)$$

$$\lambda = \frac{1}{V^2}$$

$$I_T = \gamma g L + \lambda g L (f + \sin\Theta)$$

For the sake of simplicity, let's define the parameters $\hat{\gamma}$ and $\hat{\lambda}$, equal to γg and λg:

$$\hat{\gamma} = \gamma g$$

$$\hat{\lambda} = \lambda g$$

$$I_T = L(\hat{\gamma} + \hat{\lambda}(f + \sin\Theta))$$

3.5 Relative Work Analysis

The relative works are derived in a fashion nearly identical to the relative impulses.

3.5.1 Relative Work of the Gravity Load

The ratio of the work done by the gravity load over a distance L, to the current kinetic energy of the body is found to be:

$$W_g = \frac{mg\sin\Theta L}{\frac{1}{2}mV^2} = \frac{2g\sin\Theta L}{V^2}$$

3.5.2 Relative Work of the Drag Load

The ratio of the work done by the aerodynamic drag force to the current kinetic energy of the body is found to be:

$$W_d = \frac{\frac{1}{2}\rho V^2 C_d A L}{\frac{1}{2}mV^2} = 2\hat{\gamma}L$$

Again using the previously defined constant $\hat{\gamma}$.

3.5.3 Relative Work of the Rolling Resistance

The ratio of the work done by the terrain interaction to the current kinetic energy of the body is found to be:

$$\boxed{W_r = \frac{fmgL}{\frac{1}{2}mV^2} = \frac{2fgL}{V^2}}$$

3.5.4 Summed Relative Works

Sum the relative work done by each external force, and call this the total relative work, W_T.

$$\boxed{W_T = W_d + W_g + W_r = 2\hat{\gamma}L + \frac{2g\sin\Theta L}{V^2} + \frac{2fgL}{V^2}}$$

Now recall the constant $\hat{\lambda}$, defined as follows:

$$\hat{\lambda} = \frac{g}{V^2}$$

$$W_T = 2L\left(\hat{\gamma} + \hat{\lambda}\left(f + \sin\Theta\right)\right)$$

4.0 Problem-Specific Calculations

In this section, calculations specific to the problem at hand - cross country navigation of the NavLab II mobile robot are presented. This mode of navigation is typified by the following characteristics:

1. Navigation at low speeds, typically 1 m/sec to 10 m/sec. Here I mean to imply that even a human would typically drive the vehicle at these speeds, owing to the discomfort of higher speeds.

2. Navigation across terrain whose vertical geometry rapidly changes. Off-road terrain is filled with both low frequency geometric challenges (hills and valleys) and high frequency challenges (potholes & rocks).

4.1 Force Analysis

4.1.1 Gravity Loads

The HMMWV weight is approximately 10,200 lbf, thus for every degree of pitch, the load acting upon the powertrain *increases by approximately 790 Newtons or 178 pounds.*

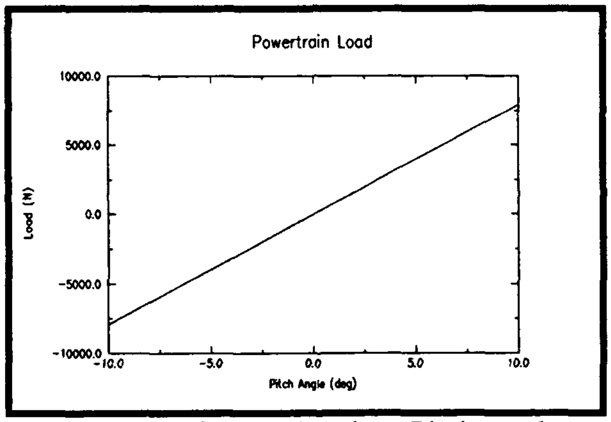

Figure 3: Gravity Load vs. Pitch Angle

The NavLab II weighs approximately five tons and has a wheelbase of approximately 3 meters. If it is assumed that the vehicle is driven along a horizontal plane and transitions to a plane inclined by 1 degree, then the following curve results. Note that for every meter / sec of vehicle speed, the powertrain load increases at a rate of approximately *265 N /sec or 60 lbf / sec.*

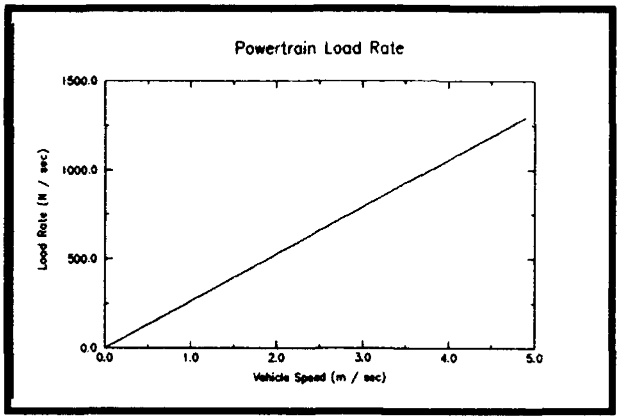

Figure 4: Gravity Load Rate vs. Vehicle Speed

4.1.2 Aerodynamic Loads

In this section, the aerodynamic forces and their time derivatives are calculated using the following numerical estimates for these constants:

$$\rho = 1.22\frac{kg}{m^3} \qquad A = 6m^2 \qquad C_d = 1.0$$

The cross-sectional area is a slight overestimate of that of the true vehicle. The drag coefficient is difficult to accurately obtain. For this analysis, I justify the use of the value 1 as follows: the nominal value of the drag coefficient for modern automobiles ranges from ~0.32 for sports cars to ~0.45 for pickup trucks. The HMMWV is not noted for its aerodynamic styling, and it has a number of appendages such as the air conditioning unit, the ERIM and the Staget that further detract from its shapely profile, thus its drag coefficient is surely larger that 0.45. An upper bound for the drag coefficient can be had from the drag coefficient for a flat plate whose surface normal is parallel to the flow, 1.95. The value of 1 is a good overestimate of a vehicle drag coefficient. Substituting these values into the given equation yields a relationship between vehicle speed and drag force:

$$\boxed{F_d = 3.66V^2}$$

The following graphs illustrate the growth of drag force as a function of velocity. The range of the first graph is 0 to 30 m / sec (0 to 67 m.p.h), the range of the second graph is 0 to 10 m / sec (0 to 22 m.p.h).[1]

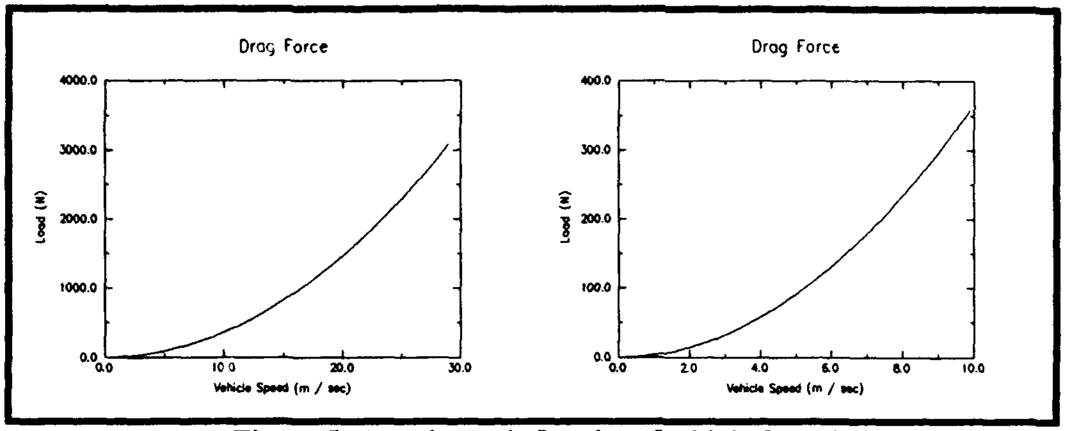

Figure 5: Aerodynamic Load vs. Vehicle Speed

It is unlikely that the HMMWV can accelerate at better than 0.3 g's. The change in drag force associated with this acceleration at 10 m/sec is 215 N / sec, (48 lbf / sec).

4.1.3 Rolling Resistance Loads

For large trucks operating in moderately packed soil, f ranges from 0.15 to 0.20. This implies that the rolling resistance is of the order of *2000 lbf*.

4.2 Relative Strength Analysis

4.2.1 Relative Strength Coefficient

In a previous section, the concept of the relative strengths of the three forces were defined. The proportionality constant among these forces was called the relative strength coefficient γ. For the case of the HMMWV, the coefficient is calculated to be:

$$\boxed{\gamma = \frac{\rho C_d A}{2mg} = \frac{(1.22\frac{kg}{m^3})\ (1)\ (6m^2)}{10200 lbf \times 4.45\,(\frac{N}{lbf})} = 1.61 \times 10^{-4}\left(\frac{s^2}{m^2}\right)}$$

This coefficient is used to determine the speed at which the aerodynamic forces approach the same order as the other forces. Recall the denominator of the relative strength ratios:

$$\boxed{f + |\sin\Theta| + \gamma V^2}$$

1. The range of the first graph corresponds approximately with that of the on-road navigation problem, while that of the second exceeds the speed range of the current off-road navigation problem.

For the domain of operation, f is approximately 0.20, while the sin term is in the range of 0 to 0.349 (0 to 20 degrees). The aerodynamic portion of the denominator approaches the same order when:

$$\gamma V^2 = 0.20$$

$$V = \sqrt{\frac{0.20}{\gamma}} = \sqrt{1242} = 35\frac{m}{s}$$

We might say that the aerodynamic forces become significant at a much lower threshold, say 10% of the rolling resistance, in which case the corresponding velocity would be 11 m/sec.

4.2.2 Gravity Relative Strength

Recall the equation for the gravity relative strength:

$$S_g = \frac{|\sin\Theta|}{f + |\sin\Theta| + \gamma V^2}$$

Using the previously determined values for f and γ, and assuming a velocity of 5 m/sec, the equation reduces to:

$$S_g = \frac{|\sin\Theta|}{0.2 + |\sin\Theta|}$$

Plotting this equation for angles between 0 and 20 degrees:

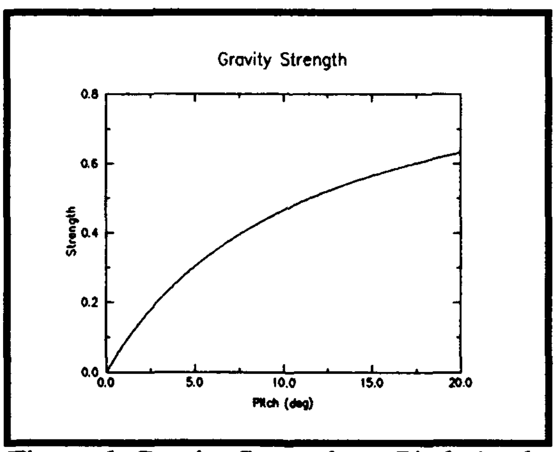

Figure 6: Gravity Strength vs. Pitch Angle

4.2.3 Aerodynamic Relative Strength

Recall the equation for the aerodynamic relative strength:

$$S_d = \frac{\gamma V^2}{f + |\sin\Theta| + \gamma V^2}$$

Using the previously selected values for f and γ, and choosing a velocity of 5 m / sec, this equation reduces to:

$$S_d = \frac{.0161}{0.21 + |\sin\Theta|}$$

Plotting this equation for angles between 0 and 20 degrees:

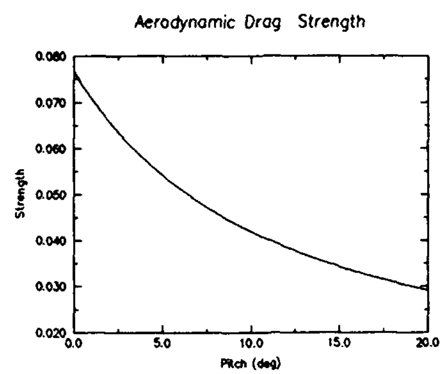

Figure 7: Aerodynamic Strength vs. Pitch

4.2.4 Rolling Friction Strength

Recall the equation for the aerodynamic relative strength:

$$S_r = \frac{f}{f + |\sin\Theta| + \gamma V^2}$$

Using the previously selected values for f and γ, and choosing a velocity of 5 m / sec, this equation reduces to:

$$S_d = \frac{0.2}{0.21 + |\sin\Theta|}$$

Plotting this equation for angles between 0 and 20 degrees:

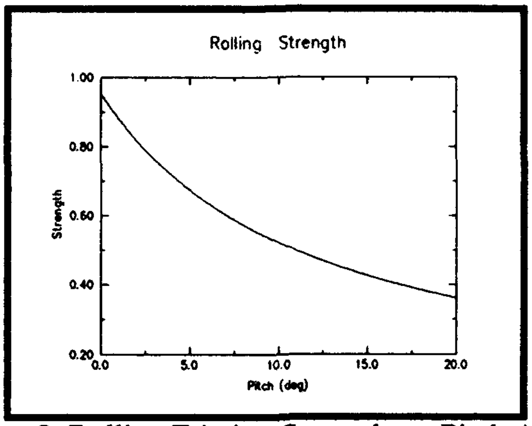

Figure 8: Rolling Friction Strength vs. Pitch Angle

4.2.5 Comparative Relative Strengths

If the three strength functions are plotted on the same axes, a direct comparison can be made:

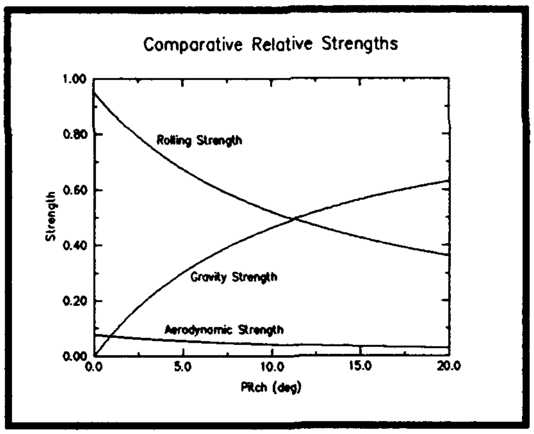

Figure 9: Comparison of Relative Strengths

Note the crossover point for rolling strength and gravity strength at approximately 10° of pitch. Under these conditions, for pitch angles less than 10° the rolling resistance dominate, for pitch angles greater than 10°, the gravity strength dominates. If the environment in which the vehicle will operate tends to be flat, then it is the case that the rolling forces always dominate the other two. Significant slopes indicate that a trade-off between dominance may occur between the two, possibly requiring that the control solution compensate for the change in dynamics.

4.3 Relative Impulse Analysis

4.3.1 Gravity Relative Impulse

Recall the gravity relative impulse equation:

$$\boxed{I_g = \frac{g \sin\Theta L}{V^2}}$$

The equation can be written on a distance-specific basis:

$$\boxed{\frac{I_g}{L} = \frac{g \sin\Theta}{V^2}}$$

The following plot contains a set of graphs of relative impulse for pitch angles between 0 and 20 degrees, at five different speeds.

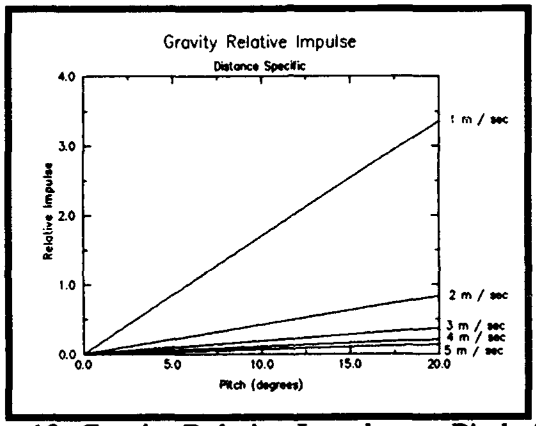

Figure 10: Gravity Relative Impulse vs. Pitch Angle

Note that when the relative impulse exceeds unity, the obstacle presents an impediment equal to all of the momentum of the vehicle. Without the addition of vehicle momentum, the vehicle cannot mount the obstacle. The relative impulse gives a quantitative measurement of the amount of additional momentum necessary to mount the obstacle.

4.3.2 Drag Force Relative Impulse

Recall the drag force relative impulse equation:

$$\boxed{I_d = \gamma g L}$$

The equation can be written on a distance-specific basis, and its constant value calculated:

$$\boxed{\frac{I_d}{L} = \gamma g = 0.001}$$

4.3.3 Rolling Resistance Relative Impulse

Recall the rolling resistance relative impulse equation:

$$\boxed{I_r = \frac{fgL}{V^2}}$$

The equation can be written on a distance-specific basis, and its constant value calculated:

$$\boxed{\frac{I_r}{L} = \frac{fg}{V^2}}$$

The following graphic plots the relative impulse as a function of the velocity of the vehicle:

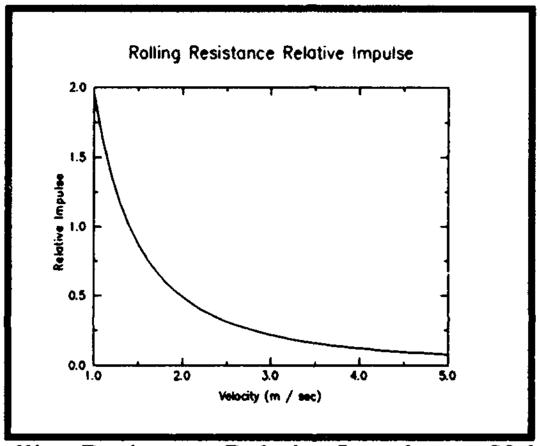

Figure 11: Rolling Resistance Relative Impulse vs. Vehicle Velocity

5.0 Discussion and Conclusions

The methods and ideas presented in this technical report have a variety of uses in the design and implementation of control systems for autonomous mobile robots. At the present time, they have found their best use in control system design. In this section, I discuss the reasoning behind the derivation and use of each concept.

5.1 Relative Force and Relative Strength

5.1.1 Meaning

I have introduced the concept of relative strength as a metric for examining the effects of the external forces on the body. One may wonder why I did not use the relative force, the ratio of the force of interest to the total force acting on the body. The reason is that the two concepts are really quite different comparisons. The relative strength compares *a force to a set of other forces*. However, the relative force compares *a force to the change in state of the body*. The total force acting on a body is, of course, found by vector addition; forces acting in opposite direction tend to cancel one another out. If one is interested in the ratio of the magnitude of a force to the magnitude of the vector sum of all of the other forces, then the relative force is useful. If, on the other hand, one wants to know the ratio of the magnitude of a force to the sum of the magnitudes of the other forces, then the relative strength is useful. I have said above that the relative force compares a force to the change in state of the body. The vector sum of the externally applied forces is that familiar left hand side of Newton's second law. In fact, it was earlier shown that this is the derivation of the relative force, hence the interpretation of it as a ratio of force to change in state.

5.1.2 Usefulness

I tend to think that the relative strength is a useful concept when attempting to divide a performance spectrum up into a set of sub-spectra, each of which is dominated by a particular force. The relative strength is an expression of the relative *magnitudes* of the forces, thus avoiding the problem of comparison at or near mechanical equilibrium. I have not yet thought of a use for the relative force in the context of this problem.

5.1.3 Limitations

The term was defined only in reference to a set of forces acting upon a body of known mass. It should be universally general to all such bodies.

5.2 Relative Impulse

5.2.1 Meaning

I have introduced the concept of relative impulse as a metric for examining the *significance* of an external force. Significance is an odd word to use in mechanics - what I mean here is that intuitively, a hill (gravity load) that is 10 miles long probably has more of an effect on a vehicle than a similar hill that is only 10 feet long. And either hill is probably more significant to a slow moving vehicle than to one that is already travelling along at 100 m.p.h. To provide a metric with a solid base in physical mechanics, I turned to the concept of *quantity of motion*, what we commonly refer to as momentum. I like the original name because it hints at the physical meaning of the term a little better. Momentum is that resistance to change of state inherent to mass, so the ratio of impulse to momentum is metric of the amount of that resistance that will be consumed (or perhaps create) as a result of an encounter with the impulse.

5.2.2 Usefulness

As an example, if the impulse of a particular hill, when compared with our current momentum yields a ratio of one half, then the hill would require us to use 50% of our momentum. This metric gives us a physical basis with which to

compare terrain obstacles. This leads to at least two potential uses. The first is in planning, in which we could choose a path that minimizes the change in momentum for a particular path. The second is in control, in which we could calculate the change in momentum that the body's propulsion system would have to create in order to negate the effects of the obstacle.

5.2.3 Limitations and Assumptions

In computing the relative momentum, I assumed a constant force F applied over a distance Δx. This allowed me to parameterize, for example, a hill in terms of its angle Θ and length L. Thus the relative momentum, as formulated in this state can only be applied under constant continuous conditions. The original integral definition would provide the canonical description. Because momentum is inherently a vector quantity, one must apply the ratio in a single dimension. This is not so much of a limitation or an assumption as a need to correctly apply a physical principle.

A second assumption that was lightly brushed over was one of the momenta of comparison. When the impulse integral is divided by the momentum there is no specific mention of which momenta we are using. The integral takes places over finite time (or even infinitesimal time) but the denominator is an instantaneous quantity. One must choose, for instance, whether to normalize by the current momentum, or a moment at the beginning of the impulse, or one somewhere in the middle. This assumption should be explicitly stated when implementing the method.

5.3 Relative Work

5.3.1 Meaning

Relative work was created for the same reason as relative impulse. It differs mainly in that it is a scalar concept. I do not yet know what practical difference this may have in application. It seems plausible that one could impart an impulse to an object without doing any work, i.e. if the object under action did not move. As I lack any concrete examples for practical application, I will simply leave the subject open for discussion.

5.4 Future Work

I am examining the usefulness of these concepts as bases for cost functions in motion planning. Optimal search algorithms may find it useful to base cost on the state of the vehicle as a means of integrating powertrain control with planning. I will also be conducting a series of test to determine the torque and power output of the HMMWV for comparison with the forces calculated in this report. Once these tests are completed, I will be designing and implementing a powertrain controller to compensate for these disturbances.

6.0 References

[1] Gillespie, T. "Fundamentals of Vehicle Dynamics" SAE Publications, 1992.

[2] Ivey, D. "Physics, Classical Mechanics and an Introduction to Statistical Mechanics," out of print. Available only through the author.